AP Live Mock Exam #1 – Question 1

For what would be accepted as work and answers for the actual AP Exam, please watch: https://bit.ly/3fq1nUM

(a) $f'(t) \cdot v_P(t) + f(t) \cdot v_P{}'(t)$

$$\frac{d}{dt}\left[f(t) \cdot v_p(t)\right]\Big|_{t=1} = f'(1) \cdot v_P(1) + f(1) \cdot v_P'(1) = f'(1) \cdot v_P(1) + f(1) \cdot a_P(1) = (2)(-29) + (1)(-10) = -68$$

(b) $$\int_0^{2.8} v_P(t)dt \approx (0.3-0)\frac{\left(v_P(0.3)+v_p(0)\right)}{2} + (1-0.3)\frac{\left(v_P(1)+v_p(0.3)\right)}{2} + (2.8-1)\frac{\left(v_P(2.8)+v_P(1)\right)}{2}$$

$$= (0.3)\frac{(55+0)}{2} + (0.7)\frac{(-29+55)}{2} + (1.8)\frac{(55+(-29))}{2} = 40.75$$

(c) $$\int_{-6}^{-2} f(t)\,dt = \int_{-6}^{5} f(t)\,dt - \int_{-2}^{5} f(t)\,dt = 7 - \int_{-2}^{5} f(t)\,dt = 7 - \left(\frac{1}{2} - \frac{1}{2} - \frac{1}{4} + \frac{1}{4} + 2 + \left(9 - \frac{1}{4}\pi(3)^2\right)\right) = -4 + \frac{9}{4}\pi$$

≈ 3.068 or 3.069

(d) $$2\int_3^5 f'(t)\,dt + \int_3^5 4\,dt = 2\big(f(5) - f(3)\big) + (4)(2) = 2(0 - \big(3 - \sqrt{5}\big) + 8 = 2 + 2\sqrt{5} \approx 6.472$$

(e) $g'(t) = f(t)$ Candidates for a continuous function on Extreme Value Theorem: Endpoints and where $g' = 0$.

t	$g(t)$
-2	0
$t\ -1$	$1/2$
$1/2$	$-1/4$
5	$11 - \frac{9}{4}\pi$

The maximum value of $g(t)$ is $g(5) = 11 - \frac{9}{4}\pi \approx 3.931$.

(f) $g'(t) = f(t)$; $g''(t) = f'(t)$

$g''(3) = f'(3) < 0$. The rate of change of g is decreasing at $t = 3$.

(g) $$\lim_{t\to 1} \frac{e^t - 3f(t)}{v_P(t) - \cos(\pi t)} = \frac{e^1 - 3f(1)}{v_P(1) - \cos(\pi 1)} = \frac{e - 3(1)}{-29 - (-1)} = \frac{3-e}{28} \approx 0.010$$

AP Live Mock Exam #1 – Question 2

(a) diameter is 6, so $r = 3$; $V = \pi(3)^2 h = 9\pi h$

$$\frac{dV}{dt} = 9\pi \frac{dh}{dt}$$

$$\left.\frac{dV}{dt}\right|_{h=10} = 9\pi\left(-\frac{1}{5}\sqrt{10}\right) = -\frac{9}{5}\pi\sqrt{10}\ \text{m}^3/\text{sec}$$

(b) asking about the rate of the rate, $\frac{dh}{dt} = -\frac{1}{5}\sqrt{h}$

$$\frac{d^2h}{dt^2} = -\frac{1}{5}\cdot\left(-\frac{1}{2}\right)h^{-\frac{1}{2}}\cdot\frac{dh}{dt} = \frac{1}{10\sqrt{h}}\left(-\frac{1}{5}\sqrt{h}\right) = \frac{1}{50} > 0$$

Since the rate of change of $\frac{dh}{dt}$ is positive for all $h > 0$, the rate of change of the height is increasing when the of height is 8 m.

(c) $$\int \frac{dh}{\sqrt{h}} = \int -\frac{1}{5}dt$$

$$2\sqrt{h} = -\frac{1}{5}t + C$$

Using $(0, 16)$ $$2\sqrt{16} = -\frac{1}{5}(0) + C$$

$$8 = C$$

$$2\sqrt{h} = -\frac{1}{5}t + 8$$

$$\sqrt{h} = -\frac{1}{10}t + 4$$

$$h = \left(-\frac{1}{10}t + 4\right)^2$$

$$h = \frac{1}{100}t^2 - \frac{4}{5}t + 16$$